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HOW SHOULD FUNDS BE ALLOCATED
TO STRENGTHEN THE STRUCTURE?

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ABSTRACT

The use of a method of optimization will help to find how to spend a given amount of money in research and industrial actions on structure for higher security concerning bird hazards.

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Abstract : The use of a method of optimization will help to find how to spend a given amount of money in research and industrial actions on structures for higher security concerning bird hazards.

Actions : 1 - The location of strikes on the plane should be tested in a blowing-engine.

2 - Fund allocation should be decided according to the following model.

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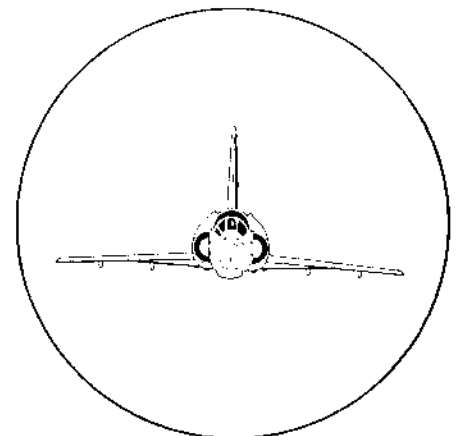
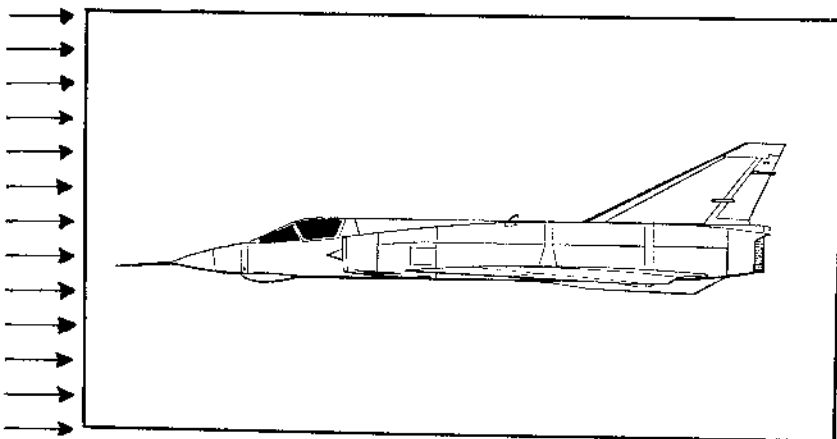
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Actions on the structure of an aircraft may be of different kinds for different effects. This paper gives a method for doing best.

1 - Location of strikes on the plane

The number of real strikes is too low to give a precise idea. So an experimental way is proposed.

A scale-model of a given type of aircraft is set in a blower-tunnel.



The air will **fly** along the scale-model under aerodynamics conditions. The bird, considered as a dead body - for the plane flies 10 to 20 times faster - will almost follow the air flow.

This will make him avoid certain parts of the plane and strike others. To have an evaluation of the probability p_j of striking part i , it is proposed to introduce a scale-model of a bird. This can be a little piece of cotton and to see where the plane is **struck**. The figure of the plane will have been divided in n parts homogeneous regarding cost and safety conditions (e.g. wings, engines, air-frame).

Then we can have the probability p_i for part i to be stroken.

$$p_i = \frac{\text{number of "birds" on part } i}{\text{number of birds on the plane}}$$

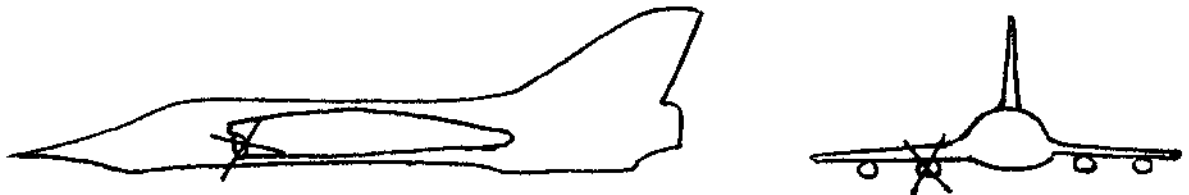
To have a general value, the test will be done several times shooting the "bird" from different points uniformly distributed on the section. The plane may be in different positions according to the different stages of flight.

2 - Probability that a shock will cause damage

When a bird strikes the plane on part i , the probability a_i that the shock causes damage will depend on the protection on this part.

$$a_i = \frac{\text{number of shocks causing damage on the part } i}{\text{number of shocks on part } i}$$

This probability can be found by a test with a "gun" towards a real airplane or from the reporting forms. The form should have a general figure of a plane on which the pilot will mark the place of the shock. With crosses



The probability of a strike on part i causes damage is then $p_i = a_i$.

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3 - Relation with cost

The cost of a damage on i , is divided in two parts.

The technical cost C_i includes the price of the new piece of equipment and the work to put it on.

The commercial cost $G \cdot t_i$ where G is the loss due to a day of immobilization and t_i the number of days of immobilization.

So the cost of damage on i is $C_i + Gt_i$.
Each value is an average value. The probability of cost due to a strike on i is then $p_i = a_i \cdot (C_i + Gt_i)$.

And the average cost due to strikes on all parts is

$$\sum_i p_i \cdot a_i \cdot (C_i + Gt_i)$$

4 - Finding the best allocation

What we want is to have the minimum average cost. This can be done by having high p_i when $a_i \cdot (C_i + Gt_i)$ is low, which means when the part i is well protected (low a_i) and the cost of the piece of equipment low (low C_i). And to have low p_i when $a_i \cdot (C_i + Gt_i)$ is high.

But it is contradictory to have good protection (a_i) for low cost (C_i). And this is why we'll explain a_i in terms of cost. A perfect protection of part i is obtained with a maximum amount of expenses in research \bar{k}_i and a maximum technical cost \bar{C}_i .

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Since the improvements have to be done on all the n airplanes of the fleet, the maximum technical cost is $n\bar{c}_i$.

Maximum means here that when $\bar{K}_i + n\bar{c}_i$ has been spent on part i , no bird will cause any damage. So more spending on part i would not bring to a better protection of this part. Otherwise, it can be determined by the maximum amount that an administration would accept to allocate to part i , assuming unlimited funds.

These maxima are rarely obtained. So let K_i and C_i be the real amounts allocated to part i for research and industrial actions. We have

$$0 \leq K_i \leq \bar{K}_i$$

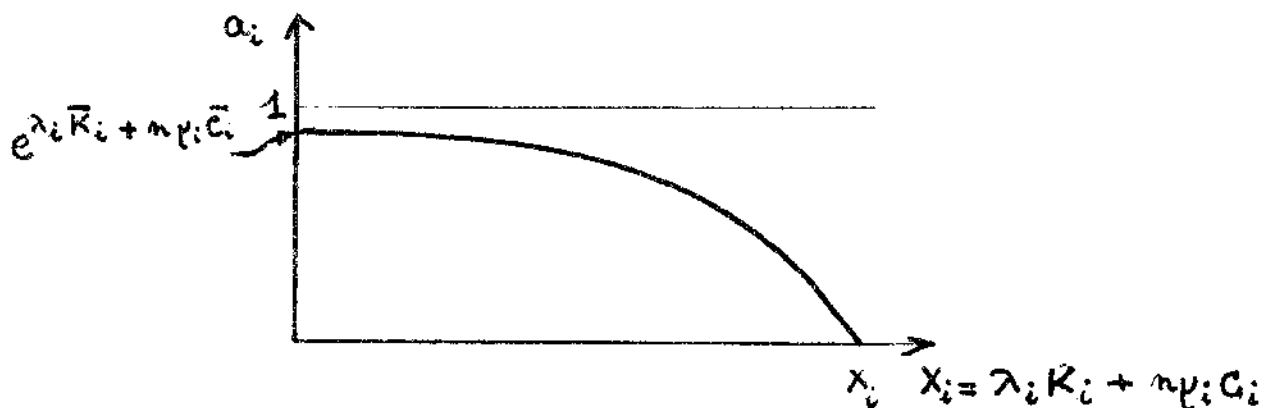
and

$$0 \leq C_i \leq \bar{C}_i$$

We now may relate the probability that a shock will do damage to the deviation between the funds allocated and the maximum useful allocations. $\bar{K}_i - K_i$ and $\bar{C}_i - C_i$

Since a pretty good protection is obtained near the maximum or for according to the part in consideration, we will introduce the coefficients λ_i and μ_i

$$a_i = e^{\lambda_i(\bar{K}_i - K_i) + n\mu_i(\bar{C}_i - C_i)} - 1$$



The problem is then to minimize $\sum p_i (e^{\lambda_i(\bar{K}_i - K_i) + \mu_i(\bar{C}_i - C_i)} - 1)(C_i + G_i)$

under budget constraints. K_M will be the total funds for research and C_M the total funds for industrial actions.

5 - The program of allocation

We may write the preceding formula in a simpler way, if we admit that a big enough amount of fund is allocated to each part i .

Let us say more than half of the maxima K_i and C_i

In this case, we have the following approximation :

$$e^{\lambda_i(\bar{K}_i - K_i) + \mu_i(\bar{C}_i - C_i)} - 1 = \lambda_i(\bar{K}_i - K_i) + \mu_i(\bar{C}_i - C_i)$$

So the program of allocation is the solution in K_i and C_i

of the following problem

$\min \sum_i p_i [\lambda_i(\bar{K}_i - K_i) + \mu_i(\bar{C}_i - C_i)](C_i + G_i)$						
<p>where</p>						
<table style="width: 100%; border: none;"> <tr> <td style="padding: 5px;">$0 \leq K_i \leq \bar{K}_i$</td> <td style="padding: 5px;">for all i</td> <td rowspan="2" style="font-size: 3em; padding: 0 10px;">}</td> <td rowspan="2" style="padding: 0 10px;">Technical constraints</td> </tr> <tr> <td style="padding: 5px;">$0 \leq C_i \leq \bar{C}_i$</td> <td style="padding: 5px;">"</td> </tr> </table>	$0 \leq K_i \leq \bar{K}_i$	for all i	}	Technical constraints	$0 \leq C_i \leq \bar{C}_i$	"
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CONCLUSION

To convince the financial administrations of a good use of funds to reduce the losses due to bird hazards, the allocation can be given after practical testings by an optimization method.

This method will lead to a compromise between research and industrial cost, depending on the part of the airplane.