BSCE 21 / WP 17
Jerusalem, 23-27 March 1992

DETERMINATION OF THE TOTAL FLYING TIME REQUIRED FOR TESTING THE PERFORMANCE OF A NEW ON-AIRFIELD BIRD STRIKE PREVENTION STRATEGY AGAINST THE STANDARD ONE

V.D. Ilyichev

Severtzov Institute
1st Kotelnichesky per., 10
Ringing Centre 109240
Moscow, Russia

V.Y. Biryukov, N.A. Nechval

Department of Control Systems
Aviation University of Riga
1, Lomonosov Street
226019 Riga
Latvia

ABSTRACT

Once it is acknowledged that bird strike statistics should be collected, it also becomes clear that the collection (and treatment) of such data only makes sense when it is done in a correct and detailed manner. This paper explores the methodological aspects of the following problems: (i) determination of the total flying time required for testing the performance of a new on-airfield bird strike prevention strategy against the standard one, (ii) comparison of two types of aircraft with respect to bird strike hazards. It is assumed that an observed series of collisions between aircraft and birds (bird strikes) is a realization of a Poisson process. An approximate test for the equality of the rate parameters of two Poisson processes is considered. The significance level, power and experiment length needed to achieve a specified power are compared to a previously studied approximate test. Both equal and unequal time intervals are taken into account. Numerical results show that this test, based on the variance stabilizing transformation, is superior in achieving nominal significance levels and powers over a wide range of parameter values and experiment lengths.

1. INTRODUCTION

Bird strike statistics are a main source of information on which the prevention of bird strike hazards should be based. Improvement of airworthiness, bird avoidance measurements, and on-airfield bird strike prevention strategies all are served by a sound knowledge about the circumstances under which bird strikes happen and the consequences of certain types of bird strikes. We emphasize one aspect of the problem.

Let us assume that an observed series of collisions between aircraft and birds (bird strikes) is a realization of a Poisson process. Denote by g the rate parameter of the Poisson process. Suppose two Poisson processes (corresponding to two series of bird strikes, respectively) with rate parameters g_1 and g_2 are observed for fixed flying times t_1 and t_2 respectively, and let x_1 and x_2 denote the number of outcomes (bird strikes) observed. Then X_1 and X_2 are Poisson random variables with means $a_1=g_1t_1$ and $a_2=g_2t_2$ respectively. Shiue and Bain (1982) proposed an approximate level α test of H_0: $g_1=g_2$ against H_1: $g_1<g_2$. Letting $d=t_2/t_1$ their test rejects H_0 when

$$S(1) = \frac{X_2-dX_1}{(d(X_1+X_2))^{1/2}} \geq z_{1-\alpha}, \qquad (1)$$

where z_q is the qth quantile of the standard normal distribution. Sichel (1973) studied S(1) and the exact significance level of the associated test for the case d=1. Shiue and Bain (1982) extended the study to general d and also investigated the power of the test. Defining $r=g_2/g_1$, they used the normal approximation to S(1) to show that for given d an appropriate choice for the value of a_1 to achieve a specified power p at a fixed alternative r is

$$a_1 = ((r/d)+1)(((1+dr)/(d+r))^{1/2}z_{1-\alpha} + z_p)^2/(r-1)^2. \qquad (2)$$

Thus if one has some estimate for the true value of g_1, then the required experiment lengths t_1 and t_2 can be approximated for given r, d, α and p.

Although the significance level and power of the test given by (1) and (2) are approximately correct, the true level is somewhat above the nominal for small d and below for large d. Also the power of the test tends to be larger than the nominal, indicating that the value of a_1 given by (2) may be larger than necessary. These results can be attributed to two factors. First, the convergence of S(1) to normality is somewhat slow. Second, in the expression for the approximate power used to derive (2), the variance of S(1) must be re-estimated, thus leading to further inaccuracies in the expression for a_1.

The test used here rejects H_0 when

$$S(2) = \frac{2((X_2+3/8)^{1/2}-(d(X_1+3/8))^{1/2})}{(1+d)^{1/2}} \geq z_{1-\alpha}. \qquad (3)$$

Since variance stabilizing transformations usually accelerate the rate of convergence to normality, $S(2)$ should converge faster than $S(1)$ as $a=\min(a_1,a_2)$ tends to infinity. Also, because the variance of $S(2)$ does not need to be estimated differently under H_1, the approximate power calculations should provide a more accurate expression for a_1. The use of the 3/8 in $S(2)$ is motivated by the results in Anscombe (1948). The mean of $S(2)$ is $2(-1+\sqrt{r})\sqrt{(a_1 d)}/\sqrt{(1+d)}$ to within $O(a-1/2)$ whether or not the 3/8 is included. The variance of $S(2)$, however, is improved from $1+O(a^{-1})$ to $1+O(a^{-2})$ by including the 3/8 terms. The exact power function of (3) is

$$\sum_{x_1=0}^{\infty}\ \sum_{x_2=[x_0]}^{\infty} \exp(-(a_1+a_2))a_1^{x_1}a_2^{x_2}/x_1!x_2!, \tag{4}$$

where $x_0=((d(x_1+3/8))^{1/2}+0.5z_{1-\alpha}(1+d)^{1/2})^2-3/8$ and $[x_0]$ is the smallest integer greater than or equal to x_0. An approximation to the power is given by

$$1-\Phi(z_{1-\alpha}-2(-1+\sqrt{r})\sqrt{(a_1 d)}/\sqrt{(1+d)}), \tag{5}$$

where Φ is the standard normal cumulative distribution function. Setting this equal to p and solving for a_1 yields

$$a_1=(z_{1-\alpha}+z_p)^2(1+d)/(4d(-1+\sqrt{r})^2). \tag{6}$$

Note that if one wishes to use a test of H_0': $g_1=g_2$ against H_1: $g_1\neq g_2$ of the level α, one may use a two-tailed exact test given by

$$X_2 \begin{cases} \in D, & \text{then } H_0' \\ \notin D, & \text{then } H_1' \end{cases} \tag{7}$$

where

$$D=\left\{x_2^*:\ p_1(o,x_1,x_2^*)\leq(1+d)^{-1}\leq p_2(o,x_1,x_2^*)\right\}, \tag{8}$$

p_1 and p_2 are given by relations

$$\sum_{i=0}^{x_2^*}\binom{x_1+x_2^*}{x_2^*-i}p_1^{x_1+i}(1-p_1)^{x_2^*-i}=\alpha/2 \tag{9}$$

and

$$\sum_{i=0}^{x_1}\binom{x_1+x_2^*}{x_2^*+i}p_2^{x_1-i}(1-p_2)^{x_2^*+i}=\alpha/2, \tag{10}$$

respectively. This test has been presented in Nechval (1982) and will not be considered further.

2. COMPARISON AND ILLUSTRATIVE EXAMPLES

Table 1 gives exact results for the two tests when $\alpha=0.05$ and $p=0.90$ for several different combinations of d and r. $\alpha(1)$ and $p(1)$ are the exact size and power of the test given by (1) and (2)

TABLE 1. Significance Level and Power Comparisons: $\alpha=0.05$, $p=0.90$

d	r	α(1)	p(1)	α(2)	p(2)
0.1	3.0	0.0591	0.9104	0.0469	0.9008
0.1	2.0	0.0574	0.9068	0.0489	0.9024
0.1	1.5	0.0535	0.9032	0.0496	0.9011
0.5	3.0	0.0538	0.9146	0.0509	0.9024
0.5	2.0	0.0524	0.9081	0.0495	0.9004
0.5	1.5	0.0515	0.9033	0.0498	0.9004
1.0	3.0	0.0494	0.9159	0.0497	0.8984
1.0	2.0	0.0494	0.9078	0.0494	0.9004
1.0	1.5	0.0498	0.9025	0.0498	0.9000
2.0	3.0	0.0408	0.9126	0.0526	0.8965
2.0	2.0	0.0466	0.9046	0.0501	0.8981
2.0	1.5	0.0485	0.9011	0.0499	0.8997
4.0	3.0	0.0383	0.9077	0.0503	0.8956
4.0	2.0	0.0441	0.9023	0.0490	0.8978
4.0	1.5	0.0472	0.9000	0.0502	0.8988

while $\alpha(2)$ and $p(2)$ are the size and power of the test given by (3) and (6). The test based on the square root transformation comes closer to the nominal level and power, particularly for the more extreme values of d. On the average the results indicate that the normal approximation to the power of the test based on S(2) is more accurate than that of the test based on S(1), and the resulting value for a_1 in (6) can in some cases save a considerable amount of observation time.

2.1. Determination of the Total Flying Time Required for Testing the Performance of a New On-airfield Bird Strike Prevention Strategy Against the Standard One

Suppose one wishes to test the performance of a new on-airfield bird strike prevention strategy in a fleet of $n_1=10$ aeroplanes against the standard on-airfield bird strike prevention strategy in another fleet of $n_2=20$ aeroplanes where $\alpha=0.05$, $p=0.90$ for $r=2$ and both fleets will be observed for $t_1=t_2=t$ flying hours. Here $d=n_2t_2/n_1t_1=2$ and equation (2) gives $a_1=g_1n_1t_1=19.5$. If the rate parameter g_1 of the new strategy is about 2 bird strikes per 100 flying hours then $t=19.5/(10\cdot0.02)=97.5$ flying hours per aeroplane and the total observation time will be 975 flying hours from the first fleet and 1950 hours from the second. By way of contrast equation (6) gives $a_1=18.7$ so that $t=93.5$ hours per aeroplane. The total flying times are 935 hours and 1870 hours, an overall savings of 120 flying hours. From Table 1 the level and power of the resulting test are 0.0501 and 0.8981, nearly exactly the nominal.

2.2. Comparison of Two Types of Aircraft with Respect to Bird Strike Hazards

To illustrate this point, we consider the following data taken from the paper (Bivings and Medve, 1990):

TABLE 2. Aircraft Strike Rate 1985-1989

type of aircraft	number of strikes	flying hours	strikes/ 100,000 hrs
T-44	157	199089	78.9
P-3	894	1237880	72.2
AV-8	112	159839	70.1
C-9	144	230123	62.6
A-6	382	708826	53.9
C-130	118	305000	38.7
F/A-18	223	644842	34.6
A-4/TA-4	295	855597	34.5
SH-60	76	227030	33.5
T-2	138	424817	32.5

Suppose one wishes to compare the aircraft types, say P-3 and T-44, with respect to bird strike hazards only. We reduce this problem to testing (at the specified level α) the null hypothesis H_0: $g_1=g_2$ against the alternative hypothesis H_1: $g_1<g_2$, where g_1 is the rate parameter of the Poisson process generated by collisions between the aircraft of type P-3 and birds, and g_2 is the rate parameter of the Poisson process generated by collisions between the aircraft of type T-44 and birds.

It follows from (3) that

$$S(2) = \frac{2((X_2+3/8)^{1/2}-(d(X_1+3/8))^{1/2})}{(1+d)^{1/2}} = 1.026 < z_{1-\alpha}=1.645, \qquad (11)$$

where X_1=894, t_1=1237880 (for P-3), X_2=157, t_2=199089 (for T-44), $d=t_2/t_1$=0.1608, and α=0.05. It results from (11) that there is no evidence to reject H_0 at the 5% level.

REFERENCES

Anscombe, F.J. (1948). The transformation of Poisson, binomial and negative-binomial data. Biometrika, vol. 35, pp. 246-254.

Bivings, B. and Medve, K.A. (1990). The U.S. Navy's bird aircraft strike hazard (BASH) problem 1985-1989. Proc. Bird Strike Committee Europe (BSCE), May 21st-25th, Helsinki, vol. 20, pp. 499-509.

Nechval, N.A. (1982). Modern Statistical Methods of Operations Research (in Russian). Riga: RCAEI.

Shiue, W. and Bain, L.J. (1982). Experiment size and power comparisons for two-sample Poisson tests. Appl. Statist., vol. 31, pp. 130-134.

Sichel, H.S. (1973). On a significance test for two Poisson variables. Appl. Statist., vol. 22, pp. 50-58.