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DETERMINATION OF NUMBER OF COLLISIONS BETWEEN AIRCRAFT AND BIRDS TO CONTROL RISKS OF ERRONEOUS JUDGMENTS ON THE BIRDSTRIKE HAZARD

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ABSTRACT

Birdstrike statistics are widely perceived as the primary instrument for monitoring the hazard and evaluating risk on individual aerodromes. However, those currently in use are not very informative and they are susceptible to variations in reporting standards. The primary goal of bird control on aerodromes should not be to prevent all birdstrikes but to minimize the likelihood of an incident that results in damage to the aircraft. A minimum acceptable standard for bird control has not been formally defined but the technique proposed in this paper forms the basis for discussion. The problem considered here is that of estimating the number of collisions between aircraft and birds to observe on each aerodrome to control the risks of erroneous judgments on the true MFTBSB (mean flying time between successive birdstrikes), and hence on the true birdstrike hazard. (It is assumed that an observed series of birdstrikes is a realization of a Poisson process.) An illustrative example is given.

1. INTRODUCTION

The research worker and statistician frequently deal with phenomena in which events of some type occur randomly in time. The Poisson process is the formal model of such phenomena.

The results of any experiment in which observation is performed continuously and "events" (i.e., occurrences of any specified kind) are tallied, can always be described by a function $x=x(t)$, which gives the number of events observed, x, during the first t units of observation, for all values of t from 0 through T, the total amount of observation performed. Such an experiment, yielding an observed function $x(t)$, is a Poisson process if the events occur randomly in the sense of the following natural definition: given that any number x of events are observed in any amount t of observation, the points of occurrence of the x events are randomly (i.e., independently uniformly) distributed between 0 and t.

An example in which the Poisson process is a very accurate and useful model is an observed series of birdstrikes.

A Poisson process can be characterized in the following two simple alternative and equivalent ways:

(a) The "waiting times" w between successive events are independently distributed with the exponential density function

$$f(w) = (1/\theta)e^{-w/\theta}\ , \quad w \geq 0. \tag{1}$$

Here θ is the mean of waiting times:

$$E(w) = \theta. \tag{2}$$

(b) The increment $y=x(t_2)-x(t_1)$ of $x(t)$ on any interval of length $l=t_2-t_1$ has the Poisson distribution

$$p(y) = e^{-gl}\,\frac{(gl)^y}{y!}\ , \quad y=0,\ 1,\ \ldots\ , \tag{3}$$

and the increments of $x(t)$ on non-overlapping intervals are independent. Here gl is the mean increment on an interval of length l. Thus g is the mean rate of occurrences, and $g=1/\theta$.

All statistical questions concerning Poisson processes involve inferences about the value of the single parameter g or θ of one process, or about the values of the respective parameters of several processes.

Methods for determining whether a given process is Poisson will not be discussed here; Lewis (1965) describes and applies several methods.

2. THE MEAN FLYING TIME BETWEEN SUCCESSIVE BIRDSTRIKES (MFTBSB)

Our model of the flying time between successive birdstrikes is given by the exponential density (it is assumed that an observed series of birdstrikes is a realization of a Poisson process):

$$f(w) = (1/\theta)e^{-w/\theta}, \quad w \geq 0, \tag{4}$$

where w is the flying time between successive birdstrikes, θ is the true, unknown MFTBSB. It is well known (Epstein and Sobel, 1953; Nechval, 1984) that if we set a risk α of rejecting the null hypothesis $\theta=\theta_a$ (the acceptable value of MFTBSB) when true, and a risk β of accepting $\theta=\theta_a$ when actually $\theta=\theta_u$ (the unacceptable value of MFTBSB), where $\theta_u < \theta_a$, then the power function of the test, or the operating characteristic curves and hence the number of collisions between aircraft and birds (birdstrikes) s may be obtained from

$$\theta_a/\theta_u = \chi^2_{1-\beta}(2s)/\chi^2_{\alpha}(2s) \tag{5}$$

where $\chi^2_{\alpha}(2s)$ is the lower α probability level and $\chi^2_{1-\beta}(2s)$ is the upper β probability level of the chi-square distribution with 2s degrees of freedom (d.f.). Thus, given θ_a, θ_u and setting the risks α and β at the levels of probability desired or at the risks one is willing to take, then by cut and try the number of required birdstrikes, s, can be determined from the tables of percentage points of the chi-square distribution.

Alternatively, to obtain an analytic solution we consider here the Wilson-Hilferty (1931) transformation of chi-square to an approximate normal variate, i.e. obtaining for r=2s d.f. the relation

$$\frac{\theta_a}{\theta_u} = \frac{r(1 - 2/(9r) + z_{1-\beta}\sqrt{2/(9r)})^3}{r(1 - 2/(9r) + z_{\alpha}\sqrt{2/(9r)})^3} \tag{6}$$

or

$$\frac{\theta_a}{\theta_u} = \frac{(1 - 1/(9s) + z_{1-\beta}\sqrt{1/(9s)})^3}{(1 - 1/(9s) + z_{\alpha}\sqrt{1/(9s)})^3} \tag{7}$$

where z_α is the lower α probability level of the standard normal distribution and $z_{1-\beta}$ is the upper β level.

With any four of the quantities θ_a, θ_u, α, β and s known, therefore, the other or fifth quantity may be found from (7). Solving (7) for s, we get

$$s = \frac{4(h-1)^2}{9(hz_\alpha - z_{1-\beta} + \sqrt{(hz_\alpha - z_{1-\beta})^2 + 4(h-1)^2})^2} = \frac{4}{9(\sqrt{c^2+4}-c)^2} \tag{8}$$

where

$$h = (\theta_a/\theta_u)^{1/3} \tag{9}$$

and

$$c = (z_{1-\beta} - hz_\alpha)/(h-1). \tag{10}$$

Note that the values of s in (8) are in close agreement with the respective values in Table II of the Epstein-Sobel (1953) paper when the values of (8) are rounded upward to the next integer. The following table gives comparisons for some selected values of α and β.

TABLE 1. Values of Number Birdstrikes, s, Required

θ_a/θ_u	α=0.01, β=0.05		α=0.05, β=0.05		α=0.10, β=0.05	
	E-S	(8)	E-S	(8)	E-S	(8)
3/2	101	98.7	67	66.3	52	51.6
2	35	34.7	23	23.0	18	17.7
5/2	21	20.3	14	13.3	11	10.2
3	15	14.4	10	9.4	8	7.2
4	10	9.4	7	6.1	5	4.6
5	8	7.2	5	4.6	4	3.5
10	4	3.9	3	2.5	2	1.8

E-S = Epstein-Sobel s (8) = s from formula (8)

3. EXAMPLE

Suppose we would like to test some fleet of airplanes to determine whether as a class they have the MFTBSB of, say, 600 flying hours or the MFTBSB of only 300 flying hours. We set a risk of 5% of rejecting the null hypothesis (MFTBSB = 600 flying hours) when true, and a risk of 10% of accepting the hypothesis (MFTBSB = 600 flying hours) when actually the true (unknown) MFTBSB is only 300 flying hours. Then our basic data would consist of the following:

$$\alpha = 0.05, \quad \beta = 0.10,$$

$$z_\alpha = -1.645, \quad z_{1-\beta} = 1.282,$$

$$\theta_a/\theta_u = 2, \quad h = 2^{1/3} = 1.2599. \tag{11}$$

From formula (8)

$$s = \frac{0.2702}{9(-3.3546+\sqrt{11.2533+0.2702})^2} = 18.7 \qquad (12)$$

or take s=19, the number of birdstrikes required for the indicated protection.

The 19 birdstrikes would be distributed among the number of representative airplanes available for test and for acceptance of the true MFTBSB = 600 flying hours we must have the observed MFTBSB

$$\hat{\theta} \geq \theta\chi^2_\alpha(2s)/(2s) = 600\chi^2_\alpha(38)/38 = 393 \text{ flying hours.} \qquad (13)$$

We reject the hypothesis that MFTBSB = 600 flying hours and accept the alternative hypothesis that MFTBSB = 300 flying hours if the observed $\hat{\theta} < 393$ flying hours.

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